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ORIGINAL ARTICLE

Comparative study of accuracy in distance measurement using: Optical and digital levels

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Abstract In this research, three levels: the optical levels **NA2** and **N3** from Leica and the digital level **SDL30** from Sokkia were subjected to distance measurement accuracy test. A base line of length 100.000 m was first established and divided into 10 equal parts using geodetic means. This was then re-measured with each of the three test levels. The mean of the distance measured by each level was compared to the geodetically established length. The r.m.s.e. values for each distance measurement were computed as standard deviations from the mean. The results showed that the Leica **N3** and **NA2** optical levels were able to measure distances to an accuracy approaching 1/5000 and 1/4000, respectively, while the **SDL30** digital level achieved a distance accuracy figure of 1/10,000. The **SDL30**, therefore, gave accuracy values in distance measurement exceeding most known tacheometric methods. The results also indicate that in the absence of distance measuring instruments, levels can be used to measure distances of 100 m range to an accuracy within 1:4000.

1. Introduction

It is well known that leveling is a main branch in geomatic engineering. It can be defined as the process of measuring vertical distances between two or more ground points either directly or indirectly for the purpose of determining their elevations. The devices designed purposely to conduct leveling are called surveying levels. The classic spirit level has a line of sight that is set horizontal by a spirit level tube. Generally, surveying levels are classified into three main types according to the method of reading the leveling rod. These are optical levels, digital levels, and laser levels.

1.1. Optical levels

These are divided into many types in accordance with the technique of obtaining coincidence between the line of collimation and the horizontal plane through the instrument. Three main types of the optical level can be distinguished. These are: (a) dumpy level, where the sighting telescope is rigidly fixed to

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the standing axis of the instrument and can be rotated in just one direction that is about the standing axis. A system of three (ideally located) leveling screws and a spirit level are used to establish a vertical standing axis and, in consequence a horizontal line of sight to enable staff readings to be taken. (b) Tilting level, where the telescope is not rigidly fixed to the standing axis, but can be tilted a small amount in the vertical plane about a pivot situated below the eyepiece of the telescope. A circular (spot) level mounted on the tribrach is usually leveled independent of the main bubble. Many designs and models of titling levels exist. Some feature coincidence bubble readers in order to increase the accuracy of setting the main bubble. (c) Automatic level, where the horizontal line of sight is established by means of a combination of optical prisms and mirrors, supported by wires as in a pendulum, the arrangement being referred to as "compensator system" (Berry, 1977; Irvine, 1988). This reduces the need to set the instrument truly level, as with the previously mentioned levels.

1.2. Digital levels

The development of these levels became possible due to advances in microchip technology and image processing. The attributes of self-leveling instrumentation coupled with digital array photography and electronic image processing have generated a digital level that is very much close to being truly automatic. The instrument is operated in conjunction with a special bar-coded staff. This type of level has the same features as automatic levels, namely the eyepiece, the focusing knob, the compensator, the circular level bubble, tangent motion, the leveling screws and objective. This is in addition to the special features pertinent to it, i.e., a built-in solid-state "camera", a storage module, a microprocessor, a display register and a control panel.

Although the operation of digital levels varies in accordance with instrument type, model and make, the procedure is to set up and level the instrument and focus it on the bar-coded staff. The operator then processes the on-off switch on the control panel to receive instructions on the display screen.

The distance to the rod can also be determined and displayed by pressing the appropriate key on the control panel. When this "measurement without recording" mode is selected, the resulting readings could be recorded manually in a field book as in dumpy, tilting and automatic optical levels.

The other mode i.e. "measuring and recording" is, however, preferable in everyday survey practice. Herein, by appropriate manipulation of the keys of the control panel, the operator enters the number and elevation of the initial benchmark on which a back sight is to be taken. The software incorporated in the instrument will display, compute and store rod readings, heights of the instrument, elevations and distances to all or some of the turning points on the line of levels. The instrument is usually capable of taking several measurements on a rod held at a point, averaging the readings and computing standard deviation of rod height readings. For more refined work, enhanced or "extended" system accuracy can be chosen. At the end of the leveling job, the memory module can be removed and interfaced to a computer where the data are downloaded and processed to give hardcopy versions of the data and the least-squares-adjusted elevations of the points occupied by the rod.

1.3. Laser levels

These are devices that emit monochromatic, intense, coherent and directional radiation in the form of a rotating beam. Basically, a laser level consists of:

(a) A laser generating and leveling mechanism which projects a horizontal laser beam, and
(b) A photo-electric laser detector. This device can be moved up and down an ordinary leveling staff to give rod readings relative to the horizontal laser plane.

Since all height measurements are related to the rotating laser beam, it is mandatory to ensure that the plane created by this beam is horizontal. In practice, this is achieved by one of three methods: either by manually using tubular bubbles and instrument foot-screws as in dumpy and tilting levels, by utilizing optical compensator system as in automatic optical levels or by using some sort of an electronically-controlled self-leveling servomotors.

Most of the laser levels recently introduced in the surveying market have either optical compensators or servomotors to achieve a horizontal laser beam.

2. Distance measurement with stadia tacheometry using levels

Distance measurement with a level is possible using the theory and techniques of stadia tacheometry. This technique uses a theodolite or level and a leveling staff. An advantage of this method is that no specialized equipment is required. It involves the use of the two short lines marked on the diaphragm of the majority of theodolite and level telescopes. These lines are called the stadia hairs or stadia lines, and are marked as in Fig. 1. The distance between the stadia hairs is fixed and is called the "stadia interval".

If observations are made to a leveling staff, the diaphragm hairs, when viewed through the instrument telescope, will appear to cover a certain length (S) of the staff, the value of S depending on the horizontal distance (D) between the instrument and staff (see Fig. 1) and is called "staff intercept". The basic principle of stadia tacheometry is shown simplified in Fig. 2.

Fig. 2 shows a vertically held leveling staff observed with a telescope of which the line of sight is inclined to the horizontal. According to the theory of tacheometry, the distance D is given by the equation:

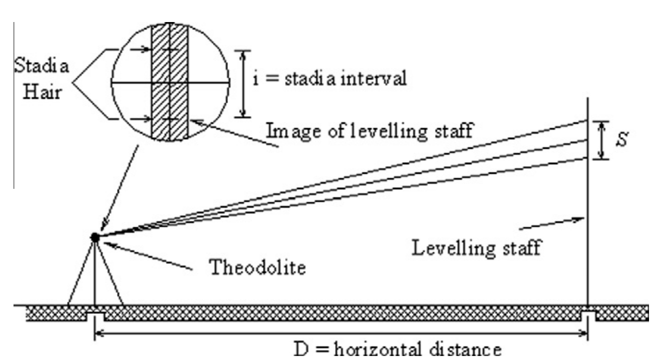

Figure 1 Stadia particulars.

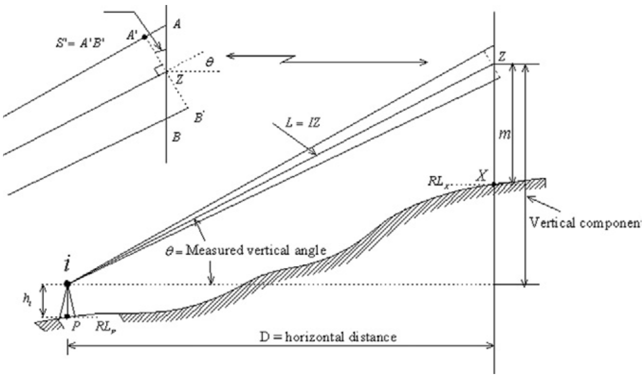

Figure 2 Principle of stadia tacheometry.

$$D = KS[\cos\theta]^2_C \text{ (Schofield, 1986).} \tag{1}$$

where K is a constant called multiplying or scale constant; θ is the angle of inclination of the line of sight, C is a constant called additive constant.

Most theodolite and level telescopes have been designed so that:

$K = 100$ and $C = 0$

For the level, the line of sight is always horizontal, i.e., $\theta = 0$.

Eq. (1) then reduces to:

$$D = KS + C \tag{2}$$

and for $K = 100$ and $C = 0$

Eq. (2) above reduces to the very simple formula:

$$D = 100\,S \tag{3}$$

3. Accuracy and sources of error in stadia measurements

The accuracy of stadia tacheometry with theodolites or levels depends on two categories of error, instrumental errors and field errors.

3.1. Instrumental errors

These include:

(a) An incorrectly assumed value for K, the multiplying constant; that is an error in the construction of the diaphragm of the instrument.
(b) Errors arising out of the assumption that modern surveying telescopes are anallactic i:e the stadia formulae (a) and (b) are always applicable, when strictly speaking both K and C are in fact variable.

The possible errors due to (a) and (b) above limit the overall accuracy of distance measurement by stadia tacheometry. In the presence of these errors the lowest accuracy of distance measurement using stadia tacheometry would be 1 = 1000, i.e. one cm error for every 10 m distance (Ali, 1995).

3.2. Field Errors

These occur from the following sources.

(i) When observing the staff, incorrect reading may be recorded which results in an error in the staff intercept, S. Assuming $K = 100$, an error of ± 1 mm in the value of S will result in an error of ± 100 mm in D. Since the staff reading accuracy decreases as D increases, the maximum length of a tacheometric sight should not be more than 120 m.
(ii) Non-verticality of the staff can be a serious source of error. This and poor accuracy of staff readings form the worst two sources of error. The error in distance due to the non-verticality of the staff is proportional to both the angle of elevation and the length of the sighting. Hence, a large error is caused by steep sightings, long sightings or a combination of both for theodolite work, and by long sightings for leveling works.

4. Purpose of the study

Having outlined the main characteristics of present day leveling instruments, a keen surveyor or civil engineer may want to know the extent to which these various instruments compare as far as the accuracy of derived distances measured is concerned and what range of applications is actually possible with each type?

The aim of this experiment is, therefore, to appraise the accuracy values with which distances can be measured using some selected leveling instruments of the types mentioned above. However, it is to be mentioned from the outset that it is not the intention of the authors of this work to endorse or recommend these or any other instruments for a certain group of applications. The authors merely attempt to evaluate, in limited and confined circumstances, the levels used in the experiment by comparing the results obtained with them with an already geodetically established 100 m base line. This will give an insight into the "relative" measuring capabilities of the instruments for limited distance measurement applications.

5. Instruments used in the test

Instruments used in the present experiment were one each of the following makes:

(1) A Leica **NA2** automatic level with a Leica (10 mm) **GPM3** parallel plate micrometer attachment and a **GPLE3** geodetic invar staff with 10 mm graduations.
(2) A Leica **N3** geodetic tilting level in conjunction with a Leica GPLE3 invar leveling.
(3) A Sokkia **SDL30** digital level used with a Sokkia BGS40 staff. The level was used in the electronic measurement mode. The rod readings were recorded manually in a field-book. No attempt was made to make use of the automatic recording and reduction module of the instrument.

Table 1 shows some of the characteristics of the three instruments believed to be of interest to the circumstances of the present experiment. Before the test commenced, all instruments were subjected to the usual series of adjustments, e.g., the two-peg test, bubble adjustment, etc. following the instructions provided by the respective manufacturers. Adjustments were carried out when deemed necessary.

Table 1 Some characteristics of the test instruments.

Characteristics	Instrument		
	Leica NA2 (automatic)	Sokkia SDL30 (digital)	Leica N3 (tilting)
Measuring range	Up to 150 m	1.6–100 m	Up to 150 m
Measuring time	Operator-dependent	4 s	Operator-dependent
Leveling accuracy (standard deviation)[a]	±0.3 mm/km	±0.6 mm/km	±0.2 mm/km
Display	Optical	LCD	Optical
Bull's eye sensitivity	8′/2 mm	8′/2 mm	8′/2 mm
Means of leveling	Automatic compensator	Automatic compensator	Split bubble
Accuracy of compensator (or bubble)	±0.3″	±0.4″	±0.2″
Display resolution	0.01 mm (on micrometer)	0.1 mm/0.01 mm (select)	0.01 mm
Telescope magnification	32×	24×	40×

[a] With invar staff and parallel plate micrometer.

6. Procedure, results and analysis

The first part of this stage consists of computing horizontal distances using Eq. (3) assuming $K = 100$ and $C = 0$. For the two optical (**NA2** and **N3**) and the one digital (**SDL30**) instruments used in the test, the discrepancies between computed distances as obtained using Eq. (3) and their known equivalents as derived from the beginning of the test using geodetic means were computed and used to derive root-mean square errors in the form of standard deviations σ_d using the standard formula:

$$\sigma_d = \pm\left[\frac{\sum w_i v_i^2}{n\sum w_i}\right]^{\frac{1}{2}} \tag{4}$$

where v_i = discrepancy between true and computed values of distance i using instrument j; n = number of acceptable distance measurement with the staff placed on a point; and w_i = a weighting function $(=\frac{1}{d_i})$.

It is to be noted that distance d_i has to be taken in meters.

For the tested levels, each distance on the test line was measured 10 times in 4 days. This gave a set of 10 measurements for each distance. These were used to compute r.m.s.e. values of distance measurement using Eq. (3). A modified form of Eq. (4) is then used to compute a grand-pooled value to represent the accuracy of the instrument in distance measurement. In this modification v_i is replaced by σ_i.

A rejection criterion was adopted in which observations showing discrepancies by more than 3σ (i.e., 99.7% confidence level) would be rejected (Schofield, 1986). It is to be noted, however, that all observations were within the criterion and no observation was rejected.

The results of this part of distance measurement test are shown in Table 2.

In the second part of this stage, a combined least squares program to solve for the two parameters K and C was written and applied to the optical levels. The derived values for K and C, based on a sample of the measured distances, are shown in Table 3 together with their standard deviation. These were used again to compute a new set of distances using Eq. (2). The discrepancies between the true and the newly-computed values were then derived and used to calculate root-mean-square errors (r.m.s.e.) using a modified form of Eq. (4) (i.e., denominator = $(n - 2)$). The results are shown in Table 3. Fig. 3 is a graphical representation of the distance accuracy test obtained with the tested instruments.

Table 2 Results of the distance measurement test using $K = 100$ and $C = 0$.

Instrument	σ_d (mm)
Leica NA2	±25
Sokkia SDL30	±11
Leica N3	±19

It is noted that the best accuracy is obtained at shorter sights. This is true for all levels used in the test. Then distance accuracy deteriorates gradually as sighting distance increases. Again, the digital level SDL30 outclassed optical instruments by almost two-fold in most cases. For all types of levels tested, the range of distance accuracy values obtained may be sufficient for a number of localized engineering surveys, such as site preparation for construction works, sewer placement and monitoring, pavement maintenance surveys etc. where errors of a few centimeters in several tens of meters are tolerable.

Finally, a worthwhile note needs to be mentioned. In this era of computerization, one major credit of any measuring equipment is its ability for automation and direct integration with other equipment for online data processing. Digital levels, such as the Sokkia SDL30 have been designed to meet this requirement. The collected data can be recorded automatically using the "record-module" resident in all digital levels. After finishing the surveying job, this data are telemetered to the base of operations, processed and made to serve as topographic layer in a geographic information system (GIS).

Results of the distance measurement accuracy test of this experiment (Tables 2 and 3) are self.-explanatory. However, it seems appropriate to supplement them with some comments. With the values of multiplication constant K and additive constant C given nominal values (i.e., $K = 100$ and $C = 0$), the horizontal accuracy values in distance measurement provided by the Leica NA2 and N3 were ±25 mm in 100 m and ±19 mm in 100 m, respectively. It is clear then that these two modern instruments (appeared in early 1990s) gave accuracy values noticeably better than what is generally believed to be attainable with stadia tacheometry, i.e., typically ±50 mm in 100 m or 1/2000 (Ali, 1995).

The Sokkia digital level SDL30 gave an accuracy figure of ±11 mm in 100 m, i.e., it excelled optical levels by almost two folds.

Table 3 Results of the distance measurement test with values of K and C computed and applied.

Instrument	σ_d (mm)	Fractional accuracy	K	C (mm)	σ_K	σ_C (mm)
Leica NA2	±22	1/4550	99.997	−0.07	±0.09	±0.005
Sokkia SDL30	±11	1/10,000	NA	NA	NA	NA
Leica N3	±18	1/5560	99.998	−0.07	±0.07	±0.006

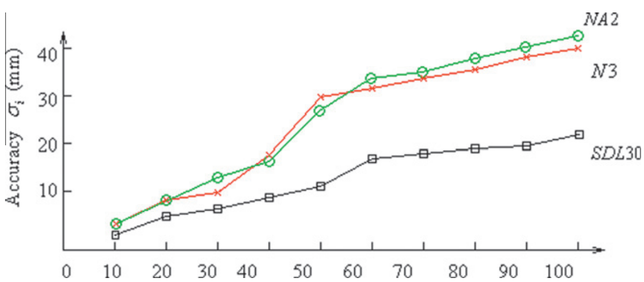

Figure 3 Accuracy of horizontal distance test.

When K and C were determined and applied for the optical levels, there was some improvement in the performance of the two optical levels. Thus corresponding values of ±22 mm (12% improvement) and ±18 mm (5% improvement) were obtained with the Leica NA2 and N3 respectively.

A number of investigators reported results of similar tests using theodolites, not levels. For distances less than 150 m, almost all investigators reported accuracy values in the range from 1/600 to 1/2100. It is clear that the horizontal accuracy figures reported in this research are much better than this, even without determining and applying K and C.

In a similar investigation using self-reduction tacheometers, Ali (1995) reported that subtense tacheometry gave the best results over 100 m distance (around 1/4700, i.e., ±21 mm in 100 m). This means that while Leica NA2 and N3 levels could produce horizontal distance accuracy figures compatible with those obtained by subtense tacheometry, the SDL30 digital level far exceeded this value (i.e., 1/10,000 compared to 1/4700). Further advantages of using levels in stadia tacheometry are, therefore, high accuracy, rapid measurement and possibility of producing contoured site plans. The distance accuracy figures shown on Table 3 are compatible with accuracy requirements for a multitude of civil engineering, cadastral, municipal, agricultural and urban planning surveys.

7. Conclusions

This experiment was carried out in order to evaluate the relative distance measurement capabilities of three different levels in distance measurement, a Leica NA2 optical level, a Leica optical geodetic level N3 and a Sokkia SDL30 digital level. For this purpose, a geodetic test line was first established on the firm flat ground of a well-protected site. The line was then re-measured using the three test instruments in turn.

With the values of multiplication constant K and additive constant C given nominal values (i.e., $K = 100$ and $C = 0$), the horizontal accuracy values in distance measurement provided by the Leica NA2 and N3 were ±25 mm in 100 m and ±19 mm in 100 m, respectively. It is clear then that these two optical instruments gave accuracy values noticeably better than what is generally believed to be attainable with stadia tacheometry (i.e., typically ±50 mm in 100 m i.e. 1/2000 [2]).

The Sokkia digital level SDL30, however, gave an accuracy figure of ±11 mm in 100 m, i.e., it excelled by almost two folds.

When K and C were determined and applied, there was some improvement in the performance of the optical levels. Thus corresponding values of ±22 mm and ±18 mm were obtained with the Leica NA2 and the Leica N3, respectively.

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